Equipped for the Future
Use Math to Solve Problems and Communicate Performance Continuum

PERFORMANCE LEVEL 1

Use Math to Solve Problems and Communicate

How adults at Level 1 **Use Math to Solve Problems and Communicate:**

- Understand, interpret, and work with pictures, numbers, and symbolic information.
- Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.
- Define and select data to be used in solving the problem.
- Determine the degree of precision required by the situation.
- Solve problem using appropriate quantitative procedures and verify that the results are reasonable.
- Communicate results using a variety of mathematical representations, including graphs, charts, tables and algebraic models.

Level 1 Indicators

Use Key Knowledge, Skills, and Strategies

Adults performing at Level 1 can:

- Read, write, and interpret very simple types of mathematical information such as
  Numbers and number sense: whole numbers (up to three digits), common monetary values, and benchmark fractions (1/2, 1/4) and percents (50%);
  Patterns/Functions/Relationships: very simple patterns, commonly-used denominations/groupings (5s, 10s, 25s), and very simple proportions (2:1, 1:2);
  Space/Shape/Measurement: high frequency standard units of measurement (pounds, feet, quarts, gallons), geometric shapes, and concepts of length and width; and
  Data/Statistics: very simple ways to interpret and represent data (checksheets, picture graphs, unambiguous bar graphs, line plots) emphasizing frequency of occurrence.
- Recall and use a few simple mathematical procedures such as very basic estimating, counting, sorting, ordering, grouping, adding on (using counting or a calculator), orally counting by 2s, 5s and 10s, and measuring length and weight using tools calibrated with whole numbers (rulers, manipulatives).
- Evaluate the degree of precision needed for the solution.
- Extract discrete information from simple and concrete data and graphs, and measure with appropriate tools, describe patterns, and/or use computational procedures effectively to solve a problem and to verify that the solution is reasonable.
- Communicate the solution to the problem orally, in role plays, with pictures, or by entries on a simple chart.

Show Fluency, Independence, and Ability to Perform in a Range of Settings

Adults performing at Level 1 can easily select and apply the knowledge, skills, and strategies at this level to independently accomplish simple, well-defined, and highly structured math tasks in one or more comfortable and familiar settings.

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Level 1 Examples of Proficient Performance

Adults performing at Level 1 can count, order, and group to accomplish a variety of goals, such as:

- Make change using the "building" method
- Determine the amount of flooring needed in a room by counting square units of material within the shape of the room
- Estimate daily/weekly wages by adding on (counting) per-hour amounts
- Fit furniture into a space by counting square units within the shape of the space or by estimating using informal measurement units
- Interpret an unambiguous horizontal or vertical bar graph in a brochure from a local clinic in order to understand information about children's health concerns
- Measure ingredients for simple recipes using benchmark fractions
- Use tallies to determine number of useable items from total number of items produced
- Sort coins into like piles, and then determine the value of each pile
- Use manipulatives, mental math, a calculator, or paper and pencil to calculate how much it will cost for 2 people to go to the movies
- Develop a schedule for how and when to take medication according to a doctor's order
- Double a recipe for chocolate chip cookies for a children's party

PERFORMANCE LEVEL 2

Use Math to Solve Problems and Communicate

How adults at Level 2 **Use Math to Solve Problems and Communicate:**

- Understand, interpret, and work with pictures, numbers, and symbolic information.
- Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.
- Define and select data to be used in solving the problem.
- Determine the degree of precision required by the situation.
- Solve problem using appropriate quantitative procedures and verify that the results are reasonable.
- Communicate results using a variety of mathematical representations, including graphs, charts, tables, and algebraic models.

Level 2 Indicators

Use Key Knowledge, Skills, and Strategies

Adults performing at Level 2 can:

- Read, write, and interpret some common types of mathematical information such as
  Numbers and number sense: whole numbers, monetary values and prices, benchmark fractions (3/4, 1/10), decimals (.25, .50, .75, .10) and percents (25%, 75%, 10%, 100%);
  Patterns/Functions/Relationships: simple patterns, probability and proportions (1:4, 4:1);
  Space/Shape/Measurement: commonly used standard units of measurement, common geometric shapes, and the concept of "area"; and
  Data/Statistics: simple ways to interpret and represent data (tables, bar graphs with and without gridlines, line graphs and pie graphs).
- Recall and use mathematic procedures such as addition and subtraction (with or without use of calculator), grouping, comparing 2 numbers, and basic estimating; and measure length, weight, and areas of standard and non-standard shapes using tools calibrated with whole numbers and benchmark equivalents (rulers, manipulatives).
- Evaluate the degree of precision needed for the solution.
- Define, select, and organize simple data, and measure with appropriate tools, describe patterns, and/or use computational procedures effectively to solve a problem and to verify that the solution is reasonable.
- Communicate the solution to the problem orally, in pictures, or in writing.

Show Fluency, Independence, and Ability to Perform in a Range of Settings

Adults performing at Level 2 can easily select and apply the knowledge, skills, and strategies at this level to independently accomplish simple, well-defined, and structured math tasks in a range of comfortable and familiar, or highly structured, settings.

Equipped for the Future
Use Math to Solve Problems and Communicate Performance Continuum

Level 2 Examples of Proficient Performance

Adults performing at Level 2 can select, record and work with simple mathematical information to accomplish a variety of goals, such as:

- Stay within a set budget while shopping for work clothes
- Determine the area of a room by grouping the square units within the shape and performing repeated addition
- Describe a simple equation for determining weekly pay based on a consistent, predictable pattern, such as $5 per hour
- Review a restaurant check for a group of 5 people to see if there are errors; estimate the sales tax on the meal
- Measure the approximate number of feet of baseboard that will be needed for a room
- Design a garden with at least 2 rectangular flowerbeds; cost out fencing and plants using a catalogue
- Survey a group on the topic of your choice and create a bar graph to display data; explain findings
- Listen to 2 weather forecasters and keep track (by graphing) of the actual high and low temperatures each day for a week to see which forecaster is more accurate
- Use a calculator to add total receipts from 6 cash registers
- Quadruple a recipe for chocolate chip cookies for a community bake sale

PERFORMANCE LEVEL 3

Use Math to Solve Problems and Communicate

How adults at Level 3 **Use Math to Solve Problems and Communicate:**

- Understand, interpret, and work with pictures, numbers, and symbolic information.
- Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.
- Define and select data to be used in solving the problem.
- Determine the degree of precision required by the situation.
- Solve problem using appropriate quantitative procedures and verify that the results are reasonable.
- Communicate results using a variety of mathematical representations, including graphs, charts, tables, and algebraic models.

Level 3 Indicators

Use Key Knowledge, Skills, and Strategies

Adults performing at Level 3 can:

- Read, write, and interpret a variety of common mathematical information such as
  Numbers and number sense: monetary values, extensions of benchmark fractions (1/8, 1/3, 1/5, etc), decimals, and percents (15%, 30%, etc.);
  Patterns/Functions/Relationships: patterns and simple formulas (such as d=rt, a=lw);
  Space/Shape/Measurement: standard units of measurement including fractional units and benchmark angle measurements (90 degrees, 360 degrees, etc), geometric shapes including shapes containing a combination of common shapes, concept of pi, and concept of converting between units of measurement; and
  Data/Statistics: ways to interpret and represent data (tables and graphs with scaling, basic statistical concepts such as range, mode, mean, and median).
- Recall and use a good store of mathematical procedures such as estimation, rounding, multiplication and division (with and without use of a calculator), adding and subtracting common fractional amounts, measure length, weight, area and circumference using tools calibrated to varying degrees of precision and converting units of measurement as appropriate.
- Evaluate the degree of precision needed for the solution.
- Define, select and organize a variety of common mathematical data and measure with appropriate tools, describe patterns, and/or use appropriate procedures effectively to solve a problem and verify that the solution is reasonable.
- Communicate the solution to the problem orally, with visual representations, in writing, by entries in a table or appropriate graph, or with basic statistics (range, mode, mean, median).

Show Fluency, Independence, and Ability to Perform in a Range of Settings

Adults performing at Level 3 can easily select and apply the knowledge, skills, and strategies at this level to independently accomplish well-defined and structured math tasks in a range of comfortable and familiar settings.

Equipped for the Future
Use Math to Solve Problems and Communicate Performance Continuum

Level 3 Examples of Proficient Performance

Adults performing at Level 3 can select, record, and work with mathematical information to accomplish a variety of goals, such as:

- Figure a tip on a restaurant bill
- Balance a checking account
- Write a children's book on multiplication facts
- Determine how much flooring is needed for a room by multiplying the length times the width of the room
- Design a survey regarding a community issue, and collect and organize the results
- Develop a monthly budget
- Design a garden to get the most space with the least amount of fencing needed
- Decide which product to buy based on a comparison of nutritional information
- Figure estimated taxes

PERFORMANCE LEVEL 4

Use Math to Solve Problems and Communicate

How adults at Level 4 **Use Math to Solve Problems and Communicate:**

- Understand, interpret, and work with pictures, numbers, and symbolic information.
- Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.
- Define and select data to be used in solving the problem.
- Determine the degree of precision required by the situation.
- Solve problem using appropriate quantitative procedures and verify that the results are reasonable.
- Communicate results using a variety of mathematical representations, including graphs, charts, tables, and algebraic models.

Level 4 Indicators

Use Key Knowledge, Skills, and Strategies
Adults performing at Level 4 can:

- Read, write, and interpret a wide variety of mathematical information such as
  Numbers and number sense: money/expenses/prices, percentages, decimals and fractions;
  Patterns/Functions/Relationships: patterns and formulas (such as $a=\pi r2$);
  Space/Shape/Measurement: units of measurement including fractional units, geometrical shapes including shapes containing a combination of common shapes, and concept of volume; and
  Data/Statistics: ways to interpret, represent and draw implications from data (graphs, tables, and simple forms of statistical analysis).
- Recall and use multi-step mathematical procedures (such as keeping accounts) that involve whole numbers as well as fractions, decimals, and/or percents, and measure volume using tools with different calibrations.
- Evaluate the degree of precision needed for the solution.
- Define, select, organize, and integrate mathematical information of different types in carrying out procedures, describing patterns, and/or measuring with appropriate tools to solve the problem and to verify that the solution is reasonable.
- Create appropriate visual or graphic representations such as charts, tables, graphs, etc. and clearly communicate the solution process and results orally or in writing to a variety of audiences.

Show Fluency, Independence, and Ability to Perform in a Range of Settings
Adults performing at Level 4 can easily select and apply the knowledge, skills, and strategies at this level to independently accomplish structured math tasks in a variety of comfortable and familiar settings.

Equipped for the Future
Use Math to Solve Problems and Communicate Performance Continuum

Level 4 Examples of Proficient Performance

Adults performing at Level 4 can select, analyze (by categorizing and comparing), and work with mathematical information to accomplish a variety of goals, such as:

- Design and measure shelves for a closet
- Keep track of monthly income and expenses
- Design a garden and determine the amount of fertilizer, fencing, and plants needed
- Develop a yearly budget and illustrate it by creating a graph
- Create a presentation using assorted charts and graphs to influence a committee
- Choose which car to buy based on information such as down payment required, monthly installments, and insurance costs
- Record product measurements on a chart to monitor whether the process is in control
- Estimate the gallons of water in an irregularly shaped garden pond in order to purchase the right amount of pond clarifier

PERFORMANCE LEVEL 5

Use Math to Solve Problems and Communicate

How adults at Level 5 **Use Math to Solve Problems and Communicate:**

- Understand, interpret, and work with pictures, numbers, and symbolic information.
- Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.
- Define and select data to be used in solving the problem.
- Determine the degree of precision required by the situation.
- Solve problem using appropriate quantitative procedures and verify that the results are reasonable.
- Communicate results using a variety of mathematical representations, including graphs, charts, tables and algebraic models.

Level 5 Indicators

Use Key Knowledge, Skills, and Strategies

Adults performing at Level 5 can:

- Read, write, and interpret a wide variety of (often) complex mathematical information such as
  Numbers and number sense: money/expenses/pricing;
  Patterns/Functions/Relationships: formulas for a variety of calculations;
  Space/Shape/Measurement: architectural symbols/ models and scale modeling; and
  Data/Statistics: ways to interpret, represent, identify trends in or draw inferences from data (complex tables and graphs; advanced forms of statistical analysis; graphing equations and generating equations from data and/or line graphs; using concept of slope).
- Research, select and apply sophisticated, multi-step mathematical concepts and procedures (such as scale modeling, cost analysis, earnings/deductions analysis)
- Evaluate the degree of precision needed for the solution.
- Independently research, select, organize and integrate mathematical information of different types in carrying out procedures, describing patterns, and/or measuring with appropriate tools, to solve the problem and to verify that the solution in reasonable.
- Create appropriate visual or graphic representations such as charts, tables, graphs, etc. and clearly communicate the solution process and results orally or in writing to a variety of audiences.

Show Fluency, Independence and Ability to Perform in a Range of Settings

Adults performing at Level 5 can easily select and apply the knowledge, skills, and strategies at this level to independently accomplish minimally structured, complex math tasks in a variety of comfortable and familiar settings.

Equipped for the Future
Use Math to Solve Problems and Communicate Performance Continuum

Level 5 Examples of Proficient Performance

Adults performing at Level 5 can select, analyze, integrate, and use mathematical information to accomplish a variety of goals, such as:

- Analyze effects of deductions on earnings and projecting annual income
- Design a "dream house"
- Use an amortization table to decide whether to refinance the mortgage on a house
- Determine a budget for a grant proposal
- Design an archway or bridge to scale
- Make a decision about how to consolidate bills and credit card payments

Performance levels for the EFF Content Standard
Use Math to Solve Problems and Communicate
mapped to the NRS Educational Functioning Levels

EFF Use Math to Solve Problems and Communicate Performance Levels	*Can be used to define an exit point for the NRS ABE Educational Functioning Level …*
Use Math Performance Level 1	Beginning Basic Education
Use Math Performance Level 2	Low Intermediate Basic Education
Use Math Performance Level 3	High Intermediate Basic Education
Use Math Performance Level 4	Low Adult Secondary Education
Use Math Performance Level 5	High Adult Secondary Education

How to read the EFF Use Math to Solve Problems and Communicate Performance Continuum

The EFF Use Math to Solve Problems and Communicate Performance Continuum is a developmental description of performance on the Use Math to Solve Problems and Communicate Standard. The continuum portrays development along four dimensions: the structure and depth of knowledge, and the fluency, independence, and range of performance. The five EFF Levels described here are points along the continuum that serve as benchmarks for key stages in development and increasing ability to accomplish important activities in daily life that require adults to use math to solve problems and communicate.

The five levels of performance described in this document cover only a portion of the performance levels possible. There are aspects of math development and performance that fall below the performance described in Level 1 and there are many levels of proficiency leading toward higher levels of expertise beyond the knowledge, skills, strategies, and performance descriptions at Level 5 on the performance continuum.

The descriptions of performance at each level of the performance continuum are anchored in analysis of data on adult learner performance. They were developed by analyzing data on use of the Use Math to Solve Problems and Communicate Standard by adult learners in adult basic education programs (including adult literacy, adult ESOL, family literacy, and adult secondary education). This empirical evidence of performance on the standard went through extensive analysis by research staff and was reviewed and amended by a panel of content experts. At each step in this process, cognitive science and math theory and research was used to guide and refine the definition of performance criteria. The number of levels defined for the EFF Use Math to Solve Problems and Communicate Performance Continuum (five) was determined through analysis and review of data. Each level describes a qualitatively distinct stage in the development of proficiency on the standard. Each level builds on the previous levels. Thus, an adult who is able to perform at Level 3 also has mastered the performance on the Standard described at Levels 1 and 2.

Each performance level on the Use Math to Solve Problems and Communicate Performance Continuum is described on a single page in this document. Each page is divided into four sections.

Section 1 is the definition of the Standard. The performance-level description starts with the components of performance of the standard. These components define the content standard and they remain the same at each level of performance. This repetition serves as a reminder that the integrated skill process defined by the components of performance for each standard is constant across all levels, from novice to expert levels of performance. What changes from level to level is the growth and complexity of the underlying knowledge base and the resulting increases in fluency and independence in using the standard to accomplish an increasing range and variety of tasks.

This definition of the standard is a useful tool for communicating to adults and their teachers the essential features of the construct, or set of targeted abilities, for each standard. By making it clear how the skill process is defined (or "unmasking the construct," as described by Gitomer & Bennett, 2002), adult learners are better able to articulate their own learning goals for improving proficiency and teachers are better able to focus learning and instructional activities that build toward the goal of increasing ability to use the standard to accomplish everyday activities. Here is how the standard is defined for all performance levels of Use Math to Solve Problems and Communicate:

- Understand, interpret, and work with pictures, numbers, and symbolic information;
- Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension;
- Define and select data to be used in solving the problem;
- Determine the degree of precision required by the situation;
- Solve problem using appropriate quantitative procedures and verify that the results are reasonable; and
- Communicate results using a variety of mathematical representations, including graphs, charts, tables, and algebraic models.

This definition of the standard is a useful tool for communicating to adults and their teachers the essential features of the construct, or set of targeted abilities, for each standard. By making it clear how the skill process is defined (or "unmasking the construct," as described by Gitomer & Bennett, 2002), adult learners are better able to articulate their own learning goals for improving proficiency and teachers are better able to focus learning and instructional activities that build toward the goal of increasing ability to use the standard to accomplish everyday activities.

Section 2 is a list of key knowledge, skills and strategies that can be observed in proficient performance at that level. These are the primary indicators (or benchmarks) of the growth and organization of the knowledge base needed for proficient performance on the standard at each level. This list is thus of central importance for designing assessments to measure performance on the standard. Because the performance levels are designed primarily as guides for assessment and not as a curriculum framework, the list does not specify details of knowledge, skills, and strategies that might be studied and taught. Nonetheless, the list can serve as a way of identifying instructional objectives for each level and provides a guide for developing criteria for placement of learners in instructional levels.

Because this list of key knowledge, skills, and strategies focuses only on those features of performance that indicate qualitative changes in what a learner knows and can do, it can be used by curriculum developers and instructors to set instructional objectives for each level and to develop more detailed curricula or learning plans that will prepare learners to develop these abilities, meet the criteria, and move on to the next level in their development of expertise.

Section 3 defines the fluency, independence and ability to perform in a range of settings expected for proficient performance on the standard at each level. Together with the descriptions of key knowledge, skills, and strategies, these descriptions serve as the primary behavioral

indicators (benchmarks) of proficient performance at each level. As such, section 3 descriptions also provide a basis for designing learning, instruction and assessment that is appropriate to that level.

Section 4 of the performance level descriptions provides a short list of examples of the purposes for using math (math activities) that can be accomplished by an adult who is performing at each level. Like Sections 2 and 3, the descriptions of math activities in Section 4 are specific to each performance level. These examples of things that adults can accomplish in the real world at each level of performance are useful to adult learners and to their teachers as ways of making concrete the purpose and need for attaining increasing knowledge and proficiency in math. By making it clear what can be accomplished at each level, the descriptions of math activities in Section 3 also provide motivation for higher levels of learning. The listing of real-world accomplishments also provides guidance for selecting and designing the content for instructional materials and assessments.

Notes on the Research Base for the Use Math to Solve Problems and Communicate Standard and Performance Continuum

In recent years, researchers and educators in the United States and abroad have been moving towards a common identification of the mathematics that adults need to function well in the world, to prepare for further education or training programs, and to prepare for and function well within the workforce. While the empirical research in adults' mathematics learning still is limited, the development of the Use Math to Solve Problems and Communicate Standard and Performance Continuum has been informed by and builds upon the extensive cognitive research on children's learning of mathematics, parallel standards development for the K–12 system, and the input of the business community.

It is clear that people learn mathematics that is useful to them in many settings outside of the classroom. Frequently the ways adults perform mathematical tasks are predicated on strategies that make sense in the environment in which they were developed and demonstrate a flexibility and conceptual basis that is closely tied to the context in which they are used. However, these procedures and strategies are often quite different from the decontextualized procedures and strategies that are taught in formal schooling. For some, the informal strategies and procedures were developed and are used alongside existing school-based strategies and procedures (Lave, Murtaugh, & de la Rocha, 1984). For others who did not attend school, informal strategies and procedures were developed to meet real-world needs (Nunes, Schliemann, Carraher, 1993; Millroy, 1992).

Adult learners in the United States generally have experienced some schooling and show evidence of both school-based and informally developed procedures and strategies. However, the knowledge and skills they have typically are fragmented and patchy and may be accompanied by "buggy" algorithms, misunderstandings and negative attitudes (Ginsburg, Gal & Schuh, 1996; Ginsburg & Gal, 1997; similar findings in the United Kingdom and the Netherlands: Evans, 2000; van Groenestijn, 2001). To fill the gaps in adults' conceptual understanding and to address the limitations of their informal and formal mathematics knowledge, the Use Math to Solve Problems and Communicate Standard and Performance Continuum emphasize the following:

- the development of facility with multiple representations of mathematical concepts,
- movement from the familiar and meaningful to the less familiar,
- connections within and across mathematical procedures, and
- the reality of multiple effective strategies to achieve the same ends.

By embedding the mathematics in realistic contexts, the boundaries and limitations between formal and informal mathematics are lessened, reducing the difficulty of transferring knowledge from decontextualized instruction to application within real world contexts.

Equipped for the Future
Use Math to Solve Problems and Communicate Performance Continuum

The Performance Continuum for the EFF Use Math to Solve Problems and Communicate Standard highlights the importance of contextualized and transferable learning for adults at each level by:

- describing the fluent and independent application of the standard that is possible in a range of settings and
- providing concrete examples of that application — "illustrative activities" in which adults can apply the key math knowledge, skills and strategies on the continuum.

These examples provide guidance for planning contextualized, level-appropriate instruction that may support transfer of math knowledge to real-world adult tasks.

In addition to the issues of adults' 'patchy' knowledge, 'buggy' algorithms and negative attitudes, the adult basic education system faces the challenge of designing instruction for adults who do not remain in educational programs for predictable amounts of time. Adults' needs for 'just-in-time' learning do not match the type of mathematics instruction found in most adult education (as well as K-12) programs. This traditional instruction is based on a linear sequencing of mathematics learning: numerical computation procedures in sequence (addition, subtraction, multiplication, division, fractions, decimals, percents), then algebra, then geometry, and then data and statistics. Research in K–12 education has shown that mathematics learning benefits from the simultaneous development of algebraic reasoning, measurement and shape, and understanding of data throughout the course of instruction. This simultaneous development is called the *parallel strands* approach to instruction (National Council of Teachers of Mathematics, 1989 & 2000), an approach which is a better fit with adults' learning needs.

The definition of the EFF Use Math to Solve Problems and Communicate Standard — an integrated process of competent application of math knowledge and strategies in a meaningful context to meet an adult purpose — supports the kind of teaching and learning in parallel content areas that the cognitive science research suggests may be most effective in developing learners' ability to use mathematics to solve problems. Components of the standard definition address development of procedural fluency, conceptual understanding and strategic competence, so that ability in all these areas is the focus of instruction. The Performance Continuum for the standard provides guidance concerning the kinds of procedural knowledge, conceptual understanding, and strategic competence that are the appropriate focus for instruction at each EFF level.

Support for the Continuum and Performance Levels in Numeracy Theory and Research

Procedural Fluency, Conceptual Understanding and Strategic Competence
Cognitive science research on learning and the development of expertise suggests that the ability to use mathematics to solve problems requires not only procedural fluency, but also conceptual understanding and strategic competence (Bransford, Brown, & Cocking,

1999; National Research Council, 2001; National Assessment Governing Board, 2000; Senk & Thompson, 2003). Expertise in mathematics as well as in other contexts requires that pertinent information be organized around important concepts and ideas rather than around surface features (Scribner, 1984; Chi & Koeske, 1983; Ma, 1999). Procedural fluency and automaticity relieve cognitive overload and are goals for computation, pattern recognition, and identification of problem types as well (Simon, 1980; Bransford, Brown & Cocking, 2000).

Traditionally, mathematics instruction in the U.S. has been relatively successful at developing procedural knowledge, generally meaning computation. Recent research has focused on how conceptual understanding and strategic competence can also be developed and documented. This research suggests that instructional strategies that build on prior learning and experience, embed new ideas within meaningful contexts, use multiple representations, and provide structured opportunities for classroom discourse about alternative strategies seem to be effective for developing conceptual understanding. Multiple representations, in particular, help learners make connections and thus, meaning, by facilitating the relationship between the concrete and abstract (Markovits & Sowder, 1991; Duvall, 1999; National Research Council, 2001).

The importance of procedural fluency, conceptual understanding, and strategic competence is highlighted at all levels of the Use Math to Solve Problems and Communicate Performance Continuum. In the case of procedural fluency, the descriptors at each of the five EFF levels suggest increased complexity and sophistication of procedures available to an adult as mathematical proficiency increases.

Conceptual understanding and strategic competence as “strands” of developing math proficiency are also represented at each level of the Performance Continuum. Note, in the following definition of the components of the standard at each level, that the concept of “developing problem-solving ability” encompasses not only the increasing complexity of the mathematical information and procedures involved, but also the increasing level of reasoning or “mathematical thinking” needed. Furthermore, the final component of the definition at each level points to the critical role of multiple representations of mathematical ideas, and the ability to communicate with them, in the development of conceptual understanding.

Equipped for the Future
Use Math to Solve Problems and Communicate Performance Continuum

Excerpts from *Use Math to Solve Problems and Communicate* Performance Continuum – Levels 1-5

Level 1
Evaluate the degree of precision needed for the solution.
Extract discrete information from simple and concrete data and graphs and measure with appropriate tools, describe patterns, and/or use computational procedures effectively to solve a problem and to verify that the solution is reasonable.
Communicate the solution to the problem orally, in role plays, with pictures, or by entries on a simple chart.

Level 2
Evaluate the degree of precision needed for the solution.
Define, select, and organize simple data and measure with appropriate tools, describe patterns, and/or use computational procedures effectively to solve a problem and to verify that the solution is reasonable.
Communicate the solution to the problem orally, in pictures, or in writing.

Level 3
Evaluate the degree of precision needed for the solution.
Define, select and convert a variety of common mathematical data and measure with appropriate tools, describe patterns, and/or use appropriate procedures effectively to solve a problem and verify that the solution is reasonable.
Communicate the solution to the problem orally, with visual representations, in writing, or by entries in a chart.

Level 4
Evaluate the degree of precision needed for the solution.
Define, select, convert, and integrate mathematical information of different types in carrying out procedures, describing patterns, and/or measuring with appropriate tools to solve the problem and to verify that the solution is reasonable.
Create visual or graphic representations such as charts, tables, graphs, etc. and clearly communicate the solution process and results orally or in writing to a variety of audiences.

Level 5
Evaluate the degree of precision needed for the solution.
Independently research, select, organize, and integrate mathematical information of different types in carrying out procedures, describing patterns, and/or measuring with appropriate tools to solve the problem and to verify that the solution in reasonable.
Create visual or graphic representations such as charts, tables, graphs, etc. and clearly communicate the solution process and results orally or in writing to a variety of audiences.

Meaningful Contexts and Representations

As part of the redesign of the 1992 International Adult Literacy Survey (IALS), a numeracy survey was developed to assess the distribution of basic skills in adult populations of participating countries. The developers of the paper describing the concepts underlying the assessment recognized that mathematical ideas are embedded within contexts and may be represented in a range of ways, including objects and pictures, numbers and symbols, formulas, diagrams and maps, graphs and tables, and texts (http://www.ets.org/all/numeracy.pdf). The importance of embedding teaching math (and other skills) in meaningful contexts was also emphasized in the SCANS report (1993) and is an integral part of national adult education standards in Australia and the United Kingdom.

Parallel Content Strands

Within the United States, the National Council of Teachers of Mathematics issued a major standards document (1989, 2000) based on cognitive science research (Kilpatrick, Martin & Schifter, 2003). The Adult Numeracy Network developed a framework for adult numeracy, adapting the NCTM standards for an adult education audience (Curry, Schmitt & Waldron, 1996). Both of these documents assume parallel instruction and learning of the content strands of number, data, measurement and shape, and algebraic reasoning. The sections below discuss each of these content strands in detail.

Number and number sense

Adults come to ABE instruction with varying and patchy intuitions, procedural skills and conceptual understanding of number and computation, number sense, estimation, mental math and numerical relationships, based on their prior formal learning as well as on informal or work-related experiences. For example, Ginsburg & Gal (1997) found that among 60 adult learners of varying "grade level" achievement, as determined by standardized test scores, virtually everyone understood and could use the benchmark percent, 50%, effectively, with somewhat fewer able to use 25% effectively. Between one third and one half (depending on the task) of all learners performed better on mental and visual percent tasks than on mathematically equivalent written computation tasks. A smaller group performed better on computation tasks than on mental or visual tasks requiring more conceptual understanding and flexible problem-solving strategies. To insure that computation can be used appropriately, it must be tied to meaningful representations. Similarly, to help adults manage complex tasks that require fluent and automatic computation, adults need to see how computational algorithms might model or complement their mental mathematics.

Research with children has provided a number of indications to suggest that working from context towards more abstract understanding is beneficial, that benchmark numbers are useful, and that the mathematics "ladder" is not inviolate. Students who worked on decimal problems presented in familiar contexts made significantly more progress than those who worked on problems without a context and less able students were able to take advantage of their everyday knowledge of decimals to make sense of their problem situations (Irwin, 2001). Markovits and Sowder (1994) documented the efficacy of using benchmark numbers in conceptualizing fractions and decimals and the usefulness of making clear the relationships of topics throughout instruction. Moss and Case (1999) found that it was beneficial for fourth-grade students to begin their study of rational numbers from percent, a concept familiar to them, and then proceed to investigate decimals and then fractions — a sequence reversed from traditional instruction.

The first component of performance in the EFF Use Math to Solve Problems and Communicate standard definition accounts for the four simultaneous and connected math content strands supported by the research. Note in the italicized words below how the EFF Performance Continuum attends to numbers and number sense at each level:

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Excerpts from *Use Math to Solve Problems and Communicate* Performance Continuum – Levels 1-5

Level 1
Read, write, and interpret *very simple types of mathematical information such as whole numbers (up to three digits), common monetary values, and benchmark fractions (1/2, 1/4) and percents (50%);* very simple patterns, commonly-used denominations/groupings (5s, 10s, 25s), and very simple proportions (2:1, 1:2); high frequency standard units of measurement (pounds, feet, quarts, gallons), geometric shapes, and concepts of length and width; and very simple ways to interpret and represent data (checksheets, picture graphs, unambiguous bar graphs, line plots) emphasizing frequency of occurrence.

Level 2
Read, write, and interpret *some common types of mathematical information such as whole numbers, monetary values and prices, benchmark fractions (3/4, 1/10), decimals (.25, .50, .75, .10) and percents (25%, 75%, 10%, 100%);* simple patterns, probability and proportions (1:4, 4:1); commonly used standard units of measurement and common geometric shapes, and the concept of "area"; and simple ways to interpret and represent data (tables, bar graphs with and without gridlines, line graphs and pie graphs).

Level 3
Read, write, and interpret *a variety of common mathematical information such as monetary values, extensions of benchmark fractions (1/8, 1/3, 1/5, etc), decimals, and percents (15%, 30%, etc.);* patterns and simple formulas (such as d=rt, a=lw); standard units of measurement including fractional units and benchmark angle measurements (90 degrees, 360 degrees, etc.), geometric shapes including shapes containing a combination of common shapes, concept of pi, and concept of converting between units of measurement; and ways to interpret and represent data (tables and graphs with scaling, basic statistical concepts such as range, mode, mean and median).

Level 4
Read, write, and interpret *a wide variety of mathematical information such as money/expenses/prices, percentages, decimals and fractions;* patterns and formulas (such as $a=\pi r2$); units of measurement including fractional units, geometrical shapes including shapes containing a combination of common shapes, and concept of volume; and ways to interpret, represent, and draw implications from data (charts and tables, pie graphs, and simple forms of statistical analysis).

Level 5
Read, write, and *interpret a wide variety of (often) complex mathematical information such as money/expenses/pricing;* formulas for a variety of calculations; architectural symbols/ models and scale modeling; and ways to interpret, represent, identify trends in or draw inferences from data (complex tables and graphs, advanced forms of statistical analysis, graphing equations and generating equations from data and/or line graphs, using concept of slope).

Patterns, functions, and algebraic thinking

Algebra has been labeled the gatekeeper for success in school and at work and (Moses & Cobb, 2002; Chazan & Bethell, 1998); however many adult education learners never have the opportunity to study formal algebra because their learning time is focused on mastering computation. From the research with children (and the early reports from the EMPower Project for ABE learners at TERC, see Ready, 2001) early exposure to

algebraic concepts embedded in real world contexts with an emphasis on multiple representations would likely help adult learners prepare for and succeed with the algebra they will need for further education and work.

Many high school students who do study algebra have difficulty understanding the relationship between the symbol manipulation that is the focus of much algebra instruction and graphical or tabular representations that are meaningful to them (McCoy, 1994). Researchers are beginning to focus on the development of algebraic thinking as a continuum that can begin in early grades with the exploration of patterns and functions within concrete or realistic problem settings, leading to "progressive formalizations" (Kaput, 1995; Chazan & Yerushalmy, 2003). Crucial to this work has been an emphasis on multiple representations (including verbal, symbolic, tabular and graphical) and anchoring learning in meaningful contexts that have assisted students in communicating with each other, clarifying meaning, and making connections among concepts (Bressner, Mayer, Moseley, Brar, Curan, Reed, & Webb, 1997).

Researchers have documented the ability of third-grade students to generalize symbolically from their number manipulations and appreciate beginning algebraic notions such as the impact of equal operations to equal things and variable (Schliemann, Goodrow & Lara-Roth, 2001; Schliemann & Carraher, 2002). Brenner, Mayer, Moseley, Brar, Duran, Reed & Webb (1997) found that given opportunity to focus on multiple representations of concepts within meaningful contexts with an emphasis on solution process, middle school pre-algebra students were better able to apply their knowledge to function-based word problems than were students whose instruction focused on symbol manipulation.

Note in the italicized words below how the EFF Performance Continuum attends to patterns, functions, and algebraic thinking at each level:

Excerpts from *Use Math to Solve Problems and Communicate* Performance Continuum – Levels 1-5

Level 1
Read, write, and interpret *very simple types of mathematical information such as* whole numbers (up to three digits), common monetary values, and benchmark fractions (1/2, 1/4) and percents (50%); *very simple patterns, commonly-used denominations/groupings (5s, 10s, 25s), and very simple proportions (2:1, 1:2)*; high frequency standard units of measurement (pounds, feet, quarts, gallons), geometric shapes, and concepts of length and width; and very simple ways to interpret and represent data (checksheets, picture graphs, unambiguous bar graphs, line plots) emphasizing frequency of occurrence.

Level 2
Read, write, and interpret *some common types of mathematical information such* as whole numbers, monetary values and prices, benchmark fractions (3/4, 1/10), decimals (.25, .50, .75, .10) and percents (25%, 75%, 10%, 100%)*; simple patterns, probability and proportions (1:4, 4:1)*; commonly used standard units of measurement and common geometric shapes, and the concept of "area"; and simple ways to interpret and represent data (tables, bar graphs with and without gridlines, line graphs and pie graphs).

Level 3
Read, write, and interpret *a variety of common mathematical information such* as monetary values, extensions of

benchmark fractions (1/8, 1/3, 1/5, etc), decimals, and percents (15%, 30%, etc.); *patterns and simple formulas (such as d=rt, a=lw)*; standard units of measurement including fractional units and benchmark angle measurements (90 degrees, 360 degrees, etc.), geometric shapes including shapes containing a combination of common shapes, concept of pi, and concept of converting between units of measurement; and ways to interpret and represent data (tables and graphs with scaling, basic statistical concepts such as range, mode, mean and median).

Level 4
Read, write, and interpret *a wide variety of mathematical information such as* money/expenses/prices, percentages, decimals and fractions; *patterns and formulas (such as $a=\pi r2$)*; units of measurement including fractional units, geometrical shapes including shapes containing a combination of common shapes, and concept of volume; and ways to interpret, represent, and draw implications from data (charts and tables, pie graphs, and simple forms of statistical analysis).

Level 5
Read, write, and *interpret a wide variety of (often) complex mathematical information such as* money/expenses/pricing; *formulas for a variety of calculations*; architectural symbols/ models and scale modeling; and ways to interpret, represent, identify trends in or draw inferences from data (complex tables and graphs, advanced forms of statistical analysis, graphing equations and generating equations from data and/or line graphs, using concept of slope).

Data, data analysis and statistics

In the 21st century, adults encounter data at every corner and must make decisions based on their understanding of data and its interpretation. Conceptions of sampling, bias, appropriate display of data and meaningful summary statistics are necessary for good judgments. These concepts are immediately useful to adults but are not necessarily predicated on computational fluency or advanced mathematical abilities. Indeed, Lesh, Amit and Schorr (1997) found that middle school students, "even those who traditionally do not perform well in mathematics, can invent more powerful ideas relating to trends, averages, and graphical representations of data than their teachers ever anticipated" (p. 65).

Addressing adults' real-world need to study data, data analysis, and statistics while they are learning other mathematics content will reduce the risk that they will focus all their learning time only on computation. Indeed, the numerical aspects of this topic as well as the multiple and integrated representations of graphs, technology, tables and mathematical communication and interpretation can be interwoven seamlessly with each of the other content strands.

Friel, Curcio and Bright (2001) present a taxonomy of skills required for graph comprehension (pp. 129-132) and a complexity progression for reading and constructing data displays. These range from the simplest (picture graphs, line plots and numerical bar graphs); to stacked bar graphs, stem plots and reading pie graphs; to constructing pie graphs, histograms, box plots and line graphs (pp. 146-150).

Note in the italicized words below how the EFF Performance Continuum attends to data, data analysis and statistics at each level:

Equipped for the Future
Use Math to Solve Problems and Communicate Performance Continuum

Excerpts from *Use Math to Solve Problems and Communicate* Performance Continuum – Levels 1-5

Level 1
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Level 2
Read, write, and interpret *some common types of mathematical information such* as whole numbers, monetary values and prices, benchmark fractions (3/4, 1/10), decimals (.25, .50, .75, .10) and percents (25%, 75%, 10%, 100%)*;* simple patterns, probability and proportions (1:4, 4:1); commonly used standard units of measurement and common geometric shapes, and the concept of "area"; and *simple ways to interpret and represent data (tables, bar graphs with and without gridlines, line graphs and pie graphs).*

Level 3
Read, write, and interpret *a variety of common mathematical information such* as monetary values, extensions of benchmark fractions (1/8, 1/3, 1/5, etc), decimals, and percents (15%, 30%, etc.); patterns and simple formulas (such as d=rt, a=lw); standard units of measurement including fractional units and benchmark angle measurements (90 degrees, 360 degrees, etc.), geometric shapes including shapes containing a combination of common shapes, concept of pi, and concept of converting between units of measurement; and *ways to interpret and represent data (tables and graphs with scaling, basic statistical concepts such as range, mode, mean and median).*

Level 4
Read, write, and interpret *a wide variety of mathematical information such as* money/expenses/prices, percentages, decimals and fractions; patterns and formulas (such as $a=\pi r2$); units of measurement including fractional units, geometrical shapes including shapes containing a combination of common shapes, and concept of volume; and *ways to interpret, represent, and draw implications from data (charts and tables, pie graphs, and simple forms of statistical analysis).*

Level 5
Read, write, and *interpret a wide variety of (often) complex mathematical information such as* money/expenses/pricing; formulas for a variety of calculations; architectural symbols/ models and scale modeling; and *ways to interpret, represent, identify trends in or draw inferences from data (complex tables and graphs, advanced forms of statistical analysis, graphing equations and generating equations from data and/or line graphs, using concept of slope).*

Measurement and spatial sense

All adults need to measure using a variety of measurement tools and techniques in work settings as well as at home. Issues of estimation, precision and accuracy arise in cooking, map reading, scale drawings, building and decorating. Adult learners bring with them some experience with measurement and measurement tools and this familiarity can provide a meaningful context for explorations of shape and number.

Much of the research on measurement focuses on how children come to understand concepts of length, area, volume, and angle (Lehrer, 2003). Adult learners may be

familiar with these concepts but may not be able to use them in a functional way. The adults may need to develop skills in the mathematics underlying the use of the measurement tools (for example, rulers with sixteenths, measuring cups with thirds, the proportion of scale drawings). Adult learners in the United States often are given formula sheets with instruction on substituting numbers within the formulae. However, if they have little idea of the meaning of the formulae and, in fact, the use of formulae before developing meaningful conceptual understanding, it is possible that, like groups of middle school students, they will be less likely to develop the needed conceptual understanding (Persek and Kirshner, 2000).

Note in the italicized words below how the EFF Performance Continuum attends to measurement and spatial sense at each level:

Equipped for the Future
Use Math to Solve Problems and Communicate Performance Continuum

Excerpts from *Use Math to Solve Problems and Communicate* Performance Continuum – Levels 1-5

Level 1
Read, write, and interpret *very simple types of mathematical information such as* whole numbers (up to three digits), common monetary values, and benchmark fractions (1/2, 1/4) and percents (50%); very simple patterns, commonly-used denominations/groupings (5s, 10s, 25s), and very simple proportions (2:1, 1:2); *high frequency standard units of measurement (pounds, feet, quarts, gallons), geometric shapes, and concepts of length and width*; and very simple ways to interpret and represent data (checksheets, picture graphs, unambiguous bar graphs, line plots) emphasizing frequency of occurrence.

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Level 4
Read, write, and interpret *a wide variety of mathematical information such as* money/expenses/prices, percentages, decimals and fractions; patterns and formulas (such as $a=\pi r2$); *units of measurement including fractional units, geometrical shapes including shapes containing a combination of common shapes, and concept of volume*; and ways to interpret, represent, and draw implications from data (charts and tables, pie graphs, and simple forms of statistical analysis).

Level 5
Read, write, and *interpret a wide variety of (often) complex mathematical information such as* money/expenses/pricing; formulas for a variety of calculations; *architectural symbols/ models and scale modeling*; and ways to interpret, represent, identify trends in or draw inferences from data (complex tables and graphs, advanced forms of statistical analysis, graphing equations and generating equations from data and/or line graphs, using concept of slope).

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