Use Math to Solve Problems and Communicate

What does research say about teaching math to adults? How can EFF help me help my students with math? Where do I start?

This issue of Hot Topics focuses on the EFF standard Use Math to Solve Problems and Communicate, addresses these questions and provides examples of activities that will help teachers incorporate the use of this Standard and its five performance levels into their teaching and assessment activities.

If you are like many teachers, the EFF math standard may challenge your notions of how math is ‘supposed’ to be taught. Studies tell us that adult basic education teachers are usually not well-prepared to teach mathematics to adults who are preparing for the GED (Ward, 2000; Mullinex, 1994), and fewer than 5% are certified in mathematics (Gal and Shuh, 1994). Many HOT Topics readers may fall into these categories, and look to Equipped for the Future for resources and ideas to teach mathematics in ways that are both meaningful to the student and understandable to the teacher. The EFF math standard calls for the introduction of number sense, algebra, geometry and data and statistics at all levels of instruction rather than for the teaching only of arithmetic at the beginning levels. In this way, the standard is in line with current theories of learning and with current approaches to math instruction (NCTM, 2000). In other ways, though, the EFF math standard will be familiar, because it, like all EFF standards, shifts the emphasis from rote learning of procedures to learning for understanding and for application in contexts important for adults.

The articles in this issue provide background in mathematics learning and describe how to use the EFF Use Math to Solve Problems and Communicate standard and performance continuum as tools to help design, implement, and assess mathematics instruction. Lynda Ginsberg gives an overview of the standard and continuum and describes how teachers can use the continuum’s five performance levels. Ellen McDevitt’s article constructs an EFF math lesson. Suzanne Elston describes her experiences teaching math in an ESOL class, and Donna Curry and Beth Bingman tell one GED teacher’s story of using EFF to strengthen her math instruction.

References


Using EFF to Bring Math Research into the Adult Ed Classroom
by Lynda Ginsburg

A recent study in the United Kingdom found that large numbers of adults return to study math, not for practical purposes, but to prove to themselves that they can indeed learn the mathematics that they were unsuccessful at during their schooling. Others, particularly those at the lowest levels, are interested in improving their math skills to help them better manage daily life tasks, as preparation for entry into a training or credentialing program, and/or so they can best support their children’s learning (The Guardian, June 8, 2004). The motivations of adults in the US are certainly very similar.

See Bringing Math Research into the Classroom, pg. 2
To help adults learn the mathematics that was formerly so difficult for them, EFF looked at existing research on mathematics learning and applied it to the particular needs of adult ABE and ASE populations. While there is much research on how children of all ages learn math, unfortunately, there is currently only a small body of research available focused specifically on how adults learn this same math. An accepted summary document of math research equivalent to that of the Reading Research Working Group (Kruidenier, 2002) currently does not exist. Thus development of the EFF math standard was informed by research on how people of all ages learn (Bransford, Brown, & Cocking, 1999), how they learn math in particular (Kilpatrick, Martin, & Schifter, 2003), by utilizing the rich standards document developed by the National Council of Teachers of Mathematics (2000), and through extensive observations of adult learners.

The Standard, Use Math to Solve Problems and Communicate, includes the words “use” and “solve problems” to inform learners and teachers that math is more than numbers and computational procedures. The Standard reflects the fact that adults can actually use the mathematics they learn, and it also reflects the research-based theories of learning indicating that effective learning builds off of prior knowledge and experience that is understood and meaningful. For adults, much of that prior knowledge and experience is grounded in their everyday lives and tasks.

This Standard and the level descriptors of the performance continuum differ in important ways from many traditional curriculum materials and textbooks. Often math is thought of as a linear progression, in which students move step by step through a series of topics, starting as a blank slate and ending at a mastery level, ready to proceed on to the next topic. Each topic is assumed to “build” on the previous one. But, unlike children, adults don’t come to education as a “blank slate!” They come to study math with some previous knowledge of fractions, decimals or percents and have experiences estimating, measuring and making decisions with numerical information.

For example, we know that virtually all adults (including those in ABE) know that 50% is equal to 1/2 and they know that 50% of $10.00 is $5.00. Many adults can compute 25% of $80 with different people using different strategies. Some people mentally divide $80 in half getting $40 and then divide it in half again, getting $20. (They recognize that these familiar fractions and percents are equivalent, know that half of a half is a fourth, and can compute mentally.) Others will change 25% to a decimal and multiply, using the traditional school procedure. Still others will estimate the answer, arriving at a reasonably close amount (Ginsburg, Gal, & Schuh, 1995).

Rather than thinking of math instruction as a ladder, a better metaphor might be a math web—expanding out from the center of adults’ previous knowledge, with many connecting threads. Equipped for the Future provides tools and support for spinning this web.

The EFF level descriptors take into account the kinds of knowledge adults bring with them, giving them “credit” for what they likely know, but also as a way of

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In order to fulfill responsibilities as parents/family members, citizens/community members, and workers, adults must be able to:

**STANDARD**

**Use Math to Solve Problems and Communicate**

- Understand, interpret, and work with pictures, numbers, and symbolic information.
- Apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.
- Define and select data to be used in solving the problem.
- Determine the degree of precision required by the situation.
- Solve problem using appropriate quantitative procedures and verify that the results are reasonable.
- Communicate results using a variety of mathematical representations, including graphs, chart, tables, and algebraic models.
encouraging teachers to find out what their learners know and can do; and teachers should then build upon this knowledge when planning instruction. We all know that we are much more likely to understand and remember new information if it relates to that which we already know.

The EFF performance continuum emphasizes elementary ideas, regardless of mathematical topic, before more complex ones (for example, calculations with benchmark numbers — 50%, 25%, 1/2 — before calculations with 12.5% or 5/6). It is important for learners to feel confident about the interconnections and equivalence among concepts such as 1/2, 50%, .5 and .50 because we frequently make decisions about which form of a number is easiest to use. It is easiest to develop this flexibility when the numbers are familiar and meaningful. The traditional focus on adding, subtracting, multiplying and dividing fractions followed by a similar pattern with decimals, and then percents, does little to help learners see relationships between them or gain flexibility moving between alternative representations of the same quantity.

The identification of a sequence that progresses from elementary ideas and

What is the Use Math to Solve Problems and Communicate Performance Continuum?

Several of the articles in this issue of Hot Topics refer to the EFF Use Math to Solve Problems and Communicate Performance Continuum. The EFF Continua of Performance are multidimensional, developmental descriptions of performance on the EFF Content Standards. They allow for descriptions of performance ranging from the novice level to the expert level by identifying key features of performance at various points along the continuum.

The EFF Use Math to Solve Problems and Communicate Performance Continuum is:

**A developmental description of performance on the standard...**

This continuum is based on recent research on how people develop and use mathematics in settings outside the classroom. Thus its descriptions of adult performance on the standard look different from the kinds of linear sequencing of mathematical learning traditionally described and taught in school. The continuum describes performance in math from Level 1 (roughly corresponding to the exit point for the beginning ABE levels on the National Reporting System) to Level 5 (corresponding to the exit point for adult secondary education); in doing so, it also describes simultaneous connections within and across mathematical content and procedures, and maintains a focus on the use of multiple strategies to meet practical real-world goals using mathematics.

**A multidimensional description of performance on the standard...**

Performance levels on Use Math to Solve Problems and Communicate are defined by key features called dimensions of performance. They include:

- the structure and depth of the knowledge base;
- fluency of performance;
- independence of performance;
- range of performance.

These dimensions reflect an understanding that the development of expertise in mathematics requires the interrelated development of:

- conceptual understanding;
- the procedural knowledge of how to apply that conceptual understanding, and
- the strategic competence to choose the most appropriate strategies for accomplishing a given task.

**A set of benchmarks for the development of proficiency on the standard...**

Performance level descriptors define points along the continuum that serve as benchmarks for key stages in development on the standard and for the increasing ability of adults to accomplish important activities that require the ability to use math to solve problems and communicate.

**A set of measures based on applied skills and processes...**

The performance continuum reflects the kinds of applied knowledge and skills adults need to meet real world purposes and goals in their lives as workers, citizens, and family members.

See page 6 for the Level 2 Performance Continuum for Use Math to Solve Problems and Communicate.

This article describes the continuum and is excerpted from the EFF Assessment Resource Collection found on-line at http://eff.cls.utk.edu/assessment/index.htm

For more about the Use Math to Solve Problems and Communicate Performance Continuum and to see the entire continuum go to http://eff.cls.utk.edu/assessment/math1.htm or http://eff.cls.utk.edu/eff_docs/EFFMathPC.doc
skills through more complex ideas and skills points to the important responsibility of the teacher to assess a learner’s knowledge and skills. Because of time away from formal schooling, spotty attendance, or other factors, many adults exhibit patchy knowledge or gaps in their understanding of various concepts. Such gaps are not identified with a standardized test score and may not be readily apparent but will have an impact on successful performance. For example, a learner who does not have an understanding of the meaning of area (and multiplication) will always confuse formulas for area and perimeter. Similarly, a learner who cannot skip count with 10s and 5s will have difficulty counting money.

Two different types of “strands” flow across all the levels of the performance continuum: problem-solving and content strands. Learners’ problem-solving abilities include:

• deciding on the necessary degree of precision;
• making decisions about the appropriate mathematical tools, data or information, reasonableness, and computational procedures;
• communicating the solution(s) of the problem in understandable and useful ways.  

The gradually higher expectations of problem-solving skills reflect not only the increasing complexity of the mathematical information and procedures, but also the increasing levels of reasoning or “mathematical thinking.” As people become more independent problem solvers, they need to be able to juggle multiple representations of the underlying mathematics.

For example, if a learner wants to know if the costs of phone service have decreased over the last 10 years, the learner needs to gather the appropriate data, decide how to characterize that data (as the average total bill, as a percentage of household income, as compared to cell phone costs, etc.), and decide how to communicate findings (visually with a bar graph, circle graphs, or a line graph; verbally in a paragraph, etc.). This is a much more complex problem-solving activity than one in which the phone costs of all the members of a class are gathered and the mean, median and mode are found to decide which is the best “average” to describe current phone costs.

The content strands of 1) number and number sense; 2) data, data analysis and statistics; 3) measurement and spatial sense; and 4) patterns, functions and algebraic thinking are found in all levels of the continuum. This approach is in line with the adult education standards documents of Australia, the U.K., and the numeracy component of the international ALL survey, as well as the NCTM K-12 standards (2000). Within each of these content strands, elementary ideas are developed before more complex ones, and instructional activities can easily span more than one strand, as they do in the real world.

The EFF math standard and performance continuum promote learning and the application of mathematics within real life, meaningful contexts. For most adults, numbers rarely appear without some kind of context. But, while adults often bring years of experience using numbers for many different life purposes, their understanding of the underlying mathematics is often quite limited. Their knowledge may be closely tied to the contexts in which they learned or developed their numerical strategies. Ultimately, adults need to be able to apply their knowledge to all kinds of contexts, both familiar and new. A good example of this is many adults’ comfort and ability calculating and estimating with decimals when dealing with money but not with equivalent calculations in less familiar contexts such as population statistics or weights expressed in decimals.

Traditionally it was thought that by working with numbers outside of any context (such as workbook computational exercises with stand-alone numbers), students would be able to apply their knowledge in “real-life” contexts, but this does not seem to be true for many people. After years of schooling, many are left with disconnected ideas and procedures that have little meaning. Although lots of people know how to divide fractions (though often they can’t remember which number to invert, or whether to invert when you multiply or divide), they have difficulty when asked to estimate the answer or describe a situation in which you would have to do such a computation.

It makes more sense to be certain that adults can actually apply their knowledge in actual useful and meaningful situations. As teachers provide a rich variety of problem contexts and spend time dis-
cussing how the activities are similar and different and discussing the underlying mathematics involved, learners gain the ability to generalize their mathematics experience. Individuals gain power by being able to take the knowledge and skills they have and use them whenever they are appropriate.

Clearly, when using mathematics to solve problems, computation is only one of many necessary skills. Effective problem solving requires many decisions—among them are

- Identify the problem,
- Select or find the data or information needed for the solution,
- Determine the degree of precision needed for the situation,
- Decide which problem solving strategies to use (computation, estimation, graph, diagram, etc.),
- Decide how to present, represent, or communicate the solution,
- Decide if the solution should be verified.

Computation is only one component of numeracy and, in fact, there are many numerical problems that do not require computation at all, such as interpreting graphical or diagrammatic information. Indeed, the GED test also emphasizes problem solving, estimation, multiple representations—not just computation. In this age of calculators and computers, it is much more important to understand the meaning of operations and calculations than to just be adept at doing them.

As teachers can use the EFF Standard Use Math to Solve Problems and Communicate to guide effective instruction, they may have to change some of their instructional practices. Traditional workbooks will not be enough—real materials will be used frequently, and teachers may well have to search beyond existing textbooks and/or develop their own activities to challenge learners to solve problems and think mathematically. This type of teaching requires that the teacher facilitate class discussions and explore the meaning of the mathematics rather than just computational rules. A teacher’s effectiveness will depend on his/her own understanding of the mathematics, and thus, some teachers may have to augment their own math content knowledge.

Students find instruction based on the EFF Standard to be more engaging, more useful, and more accessible (building from what they know and can do). However, to learners, it may not resemble what they expect math class “should” look like. They will not be sitting passively listening and writing but will be expected to participate actively. They will not be answering the teacher’s questions but will be asking questions and then trying to find the answers. They will be learning to be an active participant in their own learning.

References

The Use Math to Solve Problems and Communicate Performance Continuum:
It’s Not Just for Summative Evaluations
by Donna Curry and Beth Bingman

Allison teaches in a small rural ABE/GED program in Virginia. Twelve students are enrolled in her class, and nine or ten usually attend. The class meets three evenings a week for three-hour sessions. All the students have named passing the GED as a goal and most also share concerns about getting a job (or a better job) and giving their children more opportunities. The students’ tested abilities in reading and math vary widely with most students falling within NRS levels 1 – 3.

While Allison had always tried to do some group work with her class and often brought in articles to discuss and write about, the students spent most of their class time working as individuals or small groups in workbooks, especially for math. After she attended an EFF workshop, Allison tried using the EFF Teaching/Learning cycle to structure her classes, but lack of planning time and her students’ irregular schedules made it difficult.

When she learned about the EFF Use Math to Solve Problems and Communicate performance continuum, Allison decided to use it to help her figure out some common content that her students would be able to work on together. She found that (continued on page 6)
It’s Not Just for Summative Evaluations, continued from page 5

at Performance Level 2 adults should be able to use “simple ways to interpret and represent data (tables, bar graphs, line graphs, and pie graphs).” So for several weeks she worked with her students on basic line and bar graphs. They completed an activity in which they had to decide whether to use a bar graph or line graph to best communicate the information from data. Allison looked back at the Use Math to Solve Problems and Communicate performance continuum and realized that her students had been doing some basic organization but she hadn’t really given students the opportunity to explore how different organizational strategies affect the interpretation of the data.

From the Use Math to Solve Problems and Communicate performance continuum, Allison knew that her students should be able to “independently accomplish simple, well-defined, and structured math tasks in a range of comfortable and familiar settings.” She knew that she needed to work with her students to develop an activity that began with what is familiar to students and involved numbers that are fairly easy to manipulate (“friendly” numbers rather than “messy” ones, if possible).

To determine what her students already knew about how organization of data impacts interpretation, Allison began a discussion about why data are organized as they are. She first asked whether students thought their lives are at all influenced by data. They seemed stumped at first but then they began to give examples. One student mentioned that her doctor suggested that she get a mammogram every year because of her age. Another student then shared that her doctor recommended specific dietary restrictions because of her age. The doctors were influenced by researchers’ interpretation of data.

Allison then asked whether it makes a difference how data are organized. Students, with a little prompting, were able to build on their health examples. They explained that their insurance rates are based on data because they know that they pay different rates based on their age and sex. The younger students in the class picked up the discussion by moving from health insurance to car insurance rates. They were clearly not happy that their rates are so much higher than the “older” students. Allison reeled them in a bit. She wanted them to understand that their rates are so much higher than the “older” students. Allison reeled them in a bit. She wanted them to understand that their rates are so much higher than the “older” students.

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*NOTE: What’s important for the students to understand is that key factors in their lives are based on samplings of data that is categorized and interpreted and that decisions are then made based on those interpretations. For example, life insurance policies
vary for individuals based on their ages, whether or not the individual smokes and the health of the individual, just to name a few of the factors involved. Risk factors are used to determine whether an individual is likely to live long or not.

Allison and her students discussed the underlying assumptions about how insurance rates change for different ages. While students were in some agreement about why the cost of life insurance is more expensive for someone in their 50s than in their 20s, the younger students disagreed that younger drivers have more accidents than older drivers and, therefore, should not have to pay higher insurance premiums.

Allison saw this disagreement as an opportunity for an application of data collection, organization, analysis, and decision-making. She suggested that they collect data from all the classes in the program to see whether their assumptions about accident rates of young drivers—or those of the insurance companies—were more accurate.

In order to test their assumptions, Allison and the students had to figure out a plan (See EFF Standard Plan above). They created the following list of steps:

1. Decide what question(s) we want to ask.
2. Decide who we’re going to ask.
3. Figure out how we’re going to ask everybody.
   Once the data are collected, then
4. Decide how to group the data into different age groups.
5. Count the number in each group
6. Make a graph to show how many in each group
7. Decide what our information tells us in comparison with the data from insurance companies.

Students asked other learners in the program whether they had ever had an accident. They used a simple check sheet to record their data, placing a check for Yes (I have had an accident) or No (I have not had an accident.) They also recorded the ages at which people had accidents. Students collected information from many students but decided to focus on those individuals who had had accidents. They wanted to see the ages at which individuals seemed to have accidents. They wanted to challenge the belief that younger drivers have more auto accidents than older drivers, hence the more expensive insurance rates.

Students found that there were 15 individuals who had had prior auto accidents. The ages at which they had an accident were as follows: 27, 54, 35, 47, 32, 36, 17, 21, 22, 23, 45, 53, 54, 16, 19. Allison asked them to work in pairs to organize and represent their data using whatever reasonable organizational strategy that they wanted. The students created the following graphs (on page 8).

Once the graphs were done, they shared their results and were amazed at the different representations of the same data. Allison asked how this could be. The students realized that how the data are organized does influence how the data are represented and interpreted. They are able to see that in the second chart, it appears that there are more older drivers having accidents than in the first chart, where it appears that more young drivers have accidents.

Allison wants her students to be critical thinkers so she pushed them to look beyond their graphs. She asks them to consider whether they believe their data is a representative sample. They ponder whether their data would be similar to (continued on page 8)
other samples collected in other parts of the country or in places other than adult education programs. The students realize that the sample size and representation are important factors in working with charts and graphs. They decide that they need to question information they view in graphs and not necessarily believe everything that they see represented in graphs.

As they discuss ways to group and represent their data, one student suggested they could figure the percentage that had accidents at various ages. She suggests that maybe they should look at their data and use percents, too. Several students agreed, and then added that they wanted to understand percents anyway because they see it all the time; they just can’t remember “how to do them.” This concerned Allison because she knew that according to the Use Math to Solve Problems and Communicate performance continuum her students should be able to handle percents. She thinks to herself that this is not possible because not all of her students know their multiplication facts. According to the workbook she used to follow, her students had to know multiplication and division of whole number, plus everything about fractions before they were ready to handle percents.

She returned to the Use Math to Solve Problems and Communicate performance continuum for an answer. While the performance continuum is not a curriculum framework detailing what she is to teach, it does serve to give her some ideas on the sort of strategies and skills students should have at each level. Students should be comfortable with benchmark percents. This means that she doesn’t have to teach them everything about percents. Instead, she can focus on ensuring that her students are grounded in what percents are and focus only on percents that are already a part of students’ lives.

As Allison was considering how to introduce percents and particularly whether to continue to work with the data about insurance rates or start on a different project, she looked at a copy of the EFF Teaching/Learning Cycle. She thought she might find ideas about next steps. When she read over the cycle she realized that in their work on graphs and insurance data, the class had gone through the cycle. She had taken most of the cycle steps without focusing on them: She had determined individual’s goals and the group had identified a purpose. She had designed learning activities that addressed real-life concerns and then carried out her plans, capturing evidence through observations. They had reflected on what they had learned. What she had not done was involve her students systematically in the process. She decided to engage in Step 8, determining next steps with her students. She shared her assessment of their understanding of percents, how those skills connected to their goal of passing the GED as well as their day-to-day needs, and together the group decided to continue using the data that they were familiar with to learn more about percents.

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**1. Accidents by Age**

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<thead>
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<th>Age of Drivers</th>
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**2. Accidents by Age**

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**3. Accidents by Age**

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It’s Not Just for Summative Evaluations, continued from page 7
Conducting a Math Class the EFF Way

Remember that you’re teaching learners, not lessons. This sentence is the one that describes the major difference between the traditional math lesson and an EFF math lesson. Adult educators care deeply about their students, want them to succeed, and believe that they can succeed. Yet most continue to follow the lesson templates they acquired in college classes many years ago—the templates that put teachers as the source of knowledge with students as receptacles of that knowledge. It’s a deficit model: teacher has all the knowledge, student falls short in knowledge, teacher shares knowledge, student absorbs knowledge and makes up the deficit. The focus is on completing a lesson and taking a test in which the learner’s success is measured according to how well s/he is able to remember what the teacher has said is important.

It’s a model that doesn’t take into account the fact that learners have goals beyond learning facts. It doesn’t make room for the fact that learners already have (continued on page 10)
## Conducting a Math Class the EFF Way, continued from page 9

knowledge and skills that they can apply to learning new material. It doesn’t acknowledge that teachers can learn from students. It relies heavily on the lecture format for instruction assuming a one-lecture-fits-all standard. And it wraps up with a workbook or worksheet practice of many problems that are thought to incorporate the newly-taught information. A traditional math class tends to be quiet and static.

By contrast, the EFF math lesson centers on the learner and his/her needs. Beginning by identifying learner goals, an EFF lesson proceeds by helping learners identify and apply their prior knowledge and experiences to new learning, building an opportunity for the learner “to reflect on and monitor their own developing knowledge, skills, and learning strategies.” Throughout the lesson, the teacher adjusts the learning activities to reflect changes in learner goals and emerging learning needs. An EFF math class tends to be noisy and dynamic. The table at the right describes the differences between the two kinds of lessons.

What might a lesson plan look like for an EFF math class? Here is a suggested outline of a lesson for an EFF algebra class. For purposes of illustration, assume that the class is a typical ABE/GED class, with learners of different levels of ability in the same session. They attend class two nights a week for three hours, and you alternate reading and math instruction. Most of the learners have a goal of getting the GED, with most needing that credential so they can get a job.

In the traditional lesson, the objective is for learners to master algebra word problems, and the only measure of success is a test score. Lesson content might begin with substituting letters such as x and y in very simple equations, with the difficulty of the equations increasing with each lesson. Learners would practice these substitutions on workbook pages of rows of problems. The vocabulary of algebra would be used and explained somewhat, but the focus would be on solving equations where letters are used to represent “things we don’t know.” Teacher would use phrases such as “Let x equal the number of...” and would offer simple steps to solving algebra word problems. Students would copy what the teacher wrote, practice solving problems in their workbooks or on teacher-generated sheets, and take tests to see how well they were doing. The following EFF lesson was planned using the eight-step EFF Teaching/Learning Cycle, and it reflects the EFF Math Standard. The steps in the cycle help you devise lessons that have importance to learners, beginning with setting goals based on what the learners need to learn or be able to do. The performance components of the Standard provide descriptors of what success looks like. Look at the steps in the cycle (below) and the performance descriptors of the Standard and see if you can find them in the lesson plan that follows.

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### The EFF Teaching/Learning Cycle (see page 8)

**Step 1:** Determine individual learner’s goals and purposes and his/her prior knowledge about them. Identify the Standards that will help learner achieve these goals. Identify learner knowledge of the Standards.

**Step 2:** In a group, identify a shared interest, purpose, or goal to determine the group’s prior knowledge of this topic. Identify the Standard that will help the group address the shared goal. Make clear the connection between the class focus and individual’s needs.

**Step 3:** Design a learning activity to address the real-life concerns of the learners.

**Step 4:** Develop a plan to capture evidence and report learning.

**Step 5:** Carry out the learning activity.

**Step 6:** Observe and document evidence of performance of the Standard.

**Step 7:** Evaluate and reflect on how what was learned is transferable to real-life situations.

**Step 8:** Determine next steps to help learners meet their goals. (Return to Step 1 and/or 2)

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<thead>
<tr>
<th>Traditional Math Class</th>
<th>EFF Math Class</th>
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<td><strong>What do I need to teach?</strong></td>
<td><strong>What do they need to learn?</strong></td>
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<td>Spiral—integrated topics</td>
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<tr>
<td>Goal—teacher generated</td>
<td>Goal—student generated</td>
</tr>
<tr>
<td>Based on Standards</td>
<td>Based on Standards</td>
</tr>
</tbody>
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**Lessons—teacher centered**
- Curriculum driven
- Deficit model—fill the gap
- Follow the book
- Teach to the test

**Lessons—learner centered**
- Driven by student purposes
- Identify and apply prior knowledge
- Use contextual, authentic materials
- Teach to student goals

**Individualized work**
- Worksheets
- Seat work
- Little or no interaction with each other
- Competitive

**Collaborative and cooperative learning**
- Project-based learning
- Student teams
- Sharing, comparing, explaining
- Cooperative, mutually affirming

**Teacher as expert, student as receptacle**
- Materials = workbooks
- Proof of learning = test score

**Students and teachers co-construct knowledge**
- Materials = authentic, relevant to learner
- Proof of learning = test scores as well as demonstration of the skill in a meaningful context—student or group project
Lesson content:
• Discussion to elicit specific needs for using algebra—identify a situation in which choices are made based on data.
• Information gathering activity—learners provide facts based on situations at work, at home or in the community.
• Data collection activity—interpreting information in the situation to data by converting information provided by learners.
• Mini-lesson on ways in which algebra presents—as words, as a table of values, as a graph, and as an equation.
• Teacher-led translation of one student-generated situation to a Table of Values (TOV), a graph and an equation.
• Coming back to words—reflection on student-created work to determine what stories they tell.
• Connect to the way algebra presents on the GED.

Instructional procedure:
• Focus event to activate prior knowledge, e.g., “When can you remember having to use algebra?” “Why did you have to use it?” “What did it look like?”
• Define algebraic thinking, and then ask, “When do you have to make decisions?” “How do you compare information?” Guide learners in identifying a situation at work, at home, or in the community in which they make choices based on data. Relate that need to using algebraic thinking.
• Mini-lesson on ways in which algebra presents—as words in a situation, as Tables of Values (TOV), as graphs, and as equations. Use the Styrofoam cups activity to demonstrate conversion of words to each of the other three examples: I have to buy a coffee urn for our church kitchen. I want to buy one that’s big enough so we don’t have to worry about refilling it all the time, but not so big that we have coffee left over. We use 6-ounce Styrofoam cups. How many ounces of coffee will we use for 10 cups, 20 cups, 30 cups, and 50 cups of coffee? How big an urn should I buy? Or should I change the size of the cups? How many ounces of coffee would I use for the same amounts with cups that hold 4 ounces of liquid? 8 ounces of liquid?
• After the Styrofoam cups activity, use one of the learners’ own situations in the same way. Instruct group that you will be using wait time between asking the question and getting the answer—15 seconds for people to think before someone gives the answer.
• Use guided questioning to help learners convert the words in the situation to a TOV, a graph, and an equation. “What are we comparing in this situation?” “How should we label the columns on our table of values in order to compare the elements we want to compare?” “Does everyone agree?” “What pattern do you see in the table we’ve created?” “Does anyone see a different pattern?” “Can you describe that pattern in words?” “Now, can you describe it in graph form?” “Which of the columns is the horizontal axis? Which is the vertical axis?” “Does everyone agree?” “What do we call the axes?” “Looking at the TOV and the graph, can you write a word sentence that describes the pattern?” “Can you now translate those words to symbols?”
• Small group work—assign learners to groups. (Combine according to skill? According to interests? Decide according to who is present in class.) Each group selects another student-generated situation and creates a TOV, a graph, and an equation. Circulate and offer assistance or guidance where needed.
• Return to the words—groups exchange their TOVs, graphs, and equations with another group. New group describes in
words what the three elements describe in other forms.
• Guide reflection on the lesson. What are they uncomfortable with? What do they feel they need more work on? What makes sense?
• Relate to how algebra presents on tests and in workbooks—go through some workbooks with learners and focus on some problems. In small groups ask them to describe the situation presented in the problem in words, as a TOV, as a graph, and, finally, as an equation. Compare answers by sharing with the larger group.
• More practice if necessary.

It is even more interesting writing a plan for an EFF lesson than for a traditional lesson! It isn’t hard to imagine learners finding an EFF lesson more interesting to participate in; and when we have learner interest—when learners find that what they’re learning has impact in their lives—they tend to stick around longer.

But what about the time factor? Many adult educators worry that an EFF lesson is more time-consuming than a traditional lesson. If you assume that each lesson should take only one class period to complete—the old tyranny of the bell from school days—then the EFF lesson would take longer. The focus of an EFF lesson is on understanding something in depth, and on integrating multiple skills in one lesson. So an EFF lesson might take several sessions to complete, but those sessions would be filled with discussion and sharing and learning from one another and learning math concepts so that when the formulas are long forgotten the learner can still remember what the math was all about. Does planning the lesson take longer? Maybe the first few times you do it. But if you allow the learners to be responsible for their own learning, and require them to supply the materials that are authentic to their needs, an EFF lesson should take no longer to plan.

If we return to the first sentence—remember that you’re teaching learners, not lessons—we see that the algebra class we’ve described above teaches the learners by making the lesson relate to what is important to them. Rather than impose information, it builds understanding of the underlying principles of algebraic reasoning. Students in this class are less likely to ask the standard question, “When am I ever ‘gonna’ use this?” Such a lesson requires a willingness on the part of the teacher to let go—to allow situations to evolve in which s/he is not the all-knowing source of knowledge. It means that every algebra lesson will be different, because the learners in each class will be different. As a result, it’s a much more interesting class to teach—and a much more interesting class for learners to participate in.

Using Math in the ESOL Classroom

Suzanne Poteet Elston has been a GED instructor for over 14 years and an ESOL instructor for 5 years. Her adult education program in Tennessee began employing Equipped for the Future principles and strategies two years ago following an EFF training workshop. Recognizing her ESOL students’ need to understand and use math in their daily lives, Suzanne began to focus on the Use Math to Solve Problems and Communicate Standard with her Intermediate/Advanced ESOL class. She used the EFF Teaching/Learning Cycle as she and her class explored what they did and didn’t know about using math in an English-speaking environment. As Suzanne and her students completed an eight-week unit of study they had developed to focus on their math and language needs, they discovered that while we may do math differently from one culture or language to the next, we still need and use math for the same purposes. In the following article, Suzanne examines some of the challenges and rewards she and her students discovered while Using Math to Solve Problems and Communicate in the ESOL classroom.

Teaching and learning math in the ESOL classroom offers some unique challenges. One challenge comes from the students themselves, whose math education and experience may vary widely. Another challenge often lies with the teacher, who may be uncomfortable with addressing mathematics in the ESOL classroom. In addition, the teacher may be unprepared for the differ-
ences in the way math is often taught and performed from one culture to the next. Furthermore, ESOL students themselves may not see the relevance of learning math to learning English. To meet these challenges, the ESOL teacher must approach teaching math with a learner’s attitude and an open mind, eager to discover the students’ needs and to find strategies to teach numeracy rather than math as a set of discrete computational skills. In other words, math must have real meaning in the lives of English language learners.

Teaching math to ESOL students was unprecedented in my experience, and I found myself without a lot of models to follow. Fortunately, the EFF Teaching/Learning Cycle became a familiar and reassuring guide as my students and I explored new academic territory. As we worked our way through the Cycle, we discovered how they used math in their daily lives and identified the math and language skills they needed to perform these math-related tasks. We devised our own plans for how to teach and learn these tasks, and how to evaluate our learning (for it was indeed a learning process for all of us.) The Cycle kept us on-task in what might have been an overwhelming project and kept us mindful of how the learning related to their lives.

**Diversity of Needs and Experience in ESOL Math Learners**

Some of the problems we face in teaching adult ESOL numeracy skills derive from the same complexities we face in teaching adult ESOL literacy. Students from widely disparate educational backgrounds and experience with math tend to be placed together in an ESOL class based on their **oral English proficiency**. It is not unusual to have students with university degrees placed alongside students with only a few years of formal schooling. Plus, the multi-level ESOL classroom is still more the norm than the exception, so the ESOL teacher is often dealing with varying degrees of English proficiency as well. One of the first challenges the teacher faces is determining what the students already know about a variety of math-related topics. This needs assessment is often not easily obtained since students who actually understand and use math proficiently in their own language may lack the English skills necessary to interpret and perform math-related tasks within an English language scenario. Since typical standardized assessments often test more on the students’ ability to read and understand the problem rather than on their ability to actually use math, an alternative approach to assessment may be more useful.

I found that Steps One and Two of the Teaching/Learning Cycle offered an excellent approach to discover what ESOL students know about using Math and what they want to learn. I presented my students with a variety of everyday contexts requiring an element of math; this encouraged them to talk about how they would address these situations. These contexts may focus on any number of math-related tasks such as reading a bus schedule (telling time), estimating a restaurant food bill, taxes, and tip (money, addition, percents, making change), or following a recipe (measurements). In my class, we looked at workplace contexts, such as figuring wages earned based on pay rate and hours worked, plus estimating paycheck deductions. We also talked about family contexts, like following a budget and planning for major purchases or projects. From these discussions, we were able to identify several areas that students wanted to work on.

**Recognizing the Need for Math in ESOL Learners’ Lives**

The type of math education ESOL students need may vary from simple vocabulary acquisition to learning basic math concepts and operations. This disparity of math needs may lead the teacher to believe that math education is unnecessary for some students and unapproachable for others. Yet, as I talked with my students about their need to use and understand math in their lives, I realized that their successful adaptation to life in the U.S. depended, not just on being able to perform math skills, but in being able to communicate their findings in their new language. Thus the argument for including math instruction in the ESOL curriculum became clear to me when I considered how pervasive math is in our language and everyday life. Surely no ESOL teacher would question the necessity of teaching numbers, dates, telling time, money values, and standard English measurement systems to ESOL students.

These numeracy skills are survival skills and are a part of any basic ESOL curriculum. Yet, the more complex math skills my students wanted to master and the language skills they needed to talk about them are no less important to their survival. They wanted help with everyday numeracy tasks such as comparison-shopping, estimating earnings based on time and pay rate, making choices about saving and borrowing money, and working within a budget. They wanted to understand more about workplace decisions, such as pay rates for different employees and how and why such business decisions are made. They wanted to understand more about the language of math and business so they could communicate better with their employers about pay discrepancies, pay raises, and promotions. Through our explorations and discussions, I began to recognize how complex and pervasive their needs for math were.

As my students and I worked together through Steps Three and Four of the
Teaching/Learning Cycle, we began to develop a larger contextual framework that would offer them opportunities to practice the skills and knowledge they wanted to learn. We decided to create our own imaginary business, ESOL Designs, and to go through the business cycle of interviewing and hiring employees, setting pay rates, estimating and graphing business expenses, creating and submitting a decorating design proposal, accounting for all costs and purchases associated with the proposal, figuring payroll and deductions, and determining and graphing company profits. Thus, they would be given the chance to learn more about how American business works while they used math and the language of math and business.

Learning about ESOL Students’ Math Methodologies
Perhaps one of the most surprising challenges a teacher working with ESOL students will face is learning how to cope with the variety of approaches and methods ESOL students may use to do math. The “universal language” of math may actually vary widely from one language or culture or educational background to the next. Some ESOL students use commas where we use decimals and vice versa. Some students have an entirely different way of setting up and performing basic math operations, such as division or multiplication. ESOL students may not recognize some of our common operation symbols or must be taught the words for these symbols in order to be prepared to communicate about subjects that have a mathematical element. Furthermore, like most adults with real-life survival experience, these students may have developed their own strategies for accomplishing mathematical tasks. Many of these students, though limited in formal school math, may actually be gifted math strategists.

The teacher should be prepared to accept the students’ different math procedures and to realize that demonstrating how a problem is worked according to our commonly taught algorithms may have little meaning for ESOL students who were taught to work the problem another way. Add these factors to the different levels of math ability and training likely to exist in any ESOL classroom, and the teacher has some enormous challenges to work with.

As my students and I worked through Steps Five and Six of the Teaching/Learning Cycle, we made some of these discoveries along the way. Often we had to stop in the middle of a business process to review the mathematical concepts and skills needed to perform the task. Through this process, we discovered some very talented (though sometimes unorthodox) accountants and money managers. We found that the forms and worksheets we had created for our “company” kept us on task as we collected data about materials costs and employee payroll accounting. In addition, they served as excellent rubrics to document our success at performing the related math tasks.

Addressing Mathematical Content in the ESOL Classroom
The divergence of students’ needs once again challenges the teacher to find a meaningful way to address math content with ESOL students. While English language learners may be well aware of their needs for language education, they may be less than enthusiastic about using their “English time” to develop math skills. Certainly, this concern would be justified if the teacher’s approach to teaching math is to “drill and kill” on math computation and processes. In order to make math meaningful as a necessary component of language acquisition, the teacher needs to approach math as an integrated part of language and life. Math activities should allow students to use their reasoning skills and language skills to explore how they would solve problems with a mathematical component. For example, teaching multiplication with decimals as a discrete skill will have little meaning for many ESOL students; but putting this activity in the context of a real life situation, such as approaching the boss at work about a discrepancy in a paycheck, gives students a meaningful task for using math. In this context, they can practice figuring total pay by multiplying pay rate times hours worked (or by “adding on” or using a calculator or any other workable strategy the student may have) and then communicating those results to another person.

As my students and I completed our project and examined the success of our “company,” we used Steps Seven and Eight of the Teaching/Learning Cycle to guide our reflections about the unit of study. We discussed what we had learned and how we might use that knowledge. All students thought they were better prepared, both in language and math skills, to deal with some of the problems they face in their everyday lives. I think I am better prepared to understand and meet the numeracy needs of my ESOL students.

While using math in the ESOL classroom may seem new and challenging at first, it can be ultimately rewarding for both teacher and students. Upon reflection, the ESOL teacher will recognize that learning to use math within the context of learning English is valid, necessary, and already a part of what we do in the ESOL classroom. The challenge for the ESOL teacher is to expand this learning beyond the basics of language into the building of competence in life skills. In this way, we prepare our students to better meet the challenges they will face beyond our classrooms.
Questions for Dr. HOT Topics

1. What if my students don’t know their multiplication facts?
   Students who don’t know their times tables have other strategies to compute. Adding on, doubling, using a calculator are all examples of other strategies that work for students.

2. But this [the traditional way] is how I learned math!
   We now have a much better understanding of how people learn math. Our earlier approaches to teaching math were based on the math that was needed in the 1800’s and not based on research. Just because we were able to learn math the “old” way does not mean that it works for everyone, nor is it necessarily the most effective way to teach math.

3. How can I be sure that I’m teaching what the students need to know? How do you decide what is considered prerequisite for students?
   For example, if students want to learn about fractions as ratios, don’t they have to know how to add, subtract, multiply, and divide with fractions first? The EFF Use Math to Solve Problems and Communicate performance continuum is a great first step in guiding you in what should be taught. Using the continuum helps you make decisions about what content to teach at each level—and to what extent. For example, at Level 2, students need to be able to use benchmark fractions such as 1/2, 1/4, and 3/4. That does not mean that they have to be able to add 1/2 and 1/4 and 3/4, but rather that they know how to figure out what 1/2 of something is.

4. My students don’t have a lot of time. They only want to know the shortcuts and tricks to doing math so they can pass the TABE or GED.
   You’re right. None of us has a lot of time. That’s why it’s critical that what we do teach is meaningful and stays with us long after the class is over.

   The GED and TABE tests are very different so it’s important that you (and your students) know what you’re “teaching” to. The TABE test is decontextualized computation. You can help students transfer their learning to decontextualized problems once they have had opportunities to explore content that is meaningful. The GED is much more real-world related; there isn’t the decontextualization that there is on the TABE. The numbers are actually quite “friendly” because the test assesses understanding of concepts rather than computation.

   If you use the EFF Use Math to Solve Problems and Communicate Standard and performance level descriptors, you’ll be teaching students skills that can be used on the TABE as well as the GED. In fact, at the lower levels of the continuum, you’ll see that you’re encouraged to tackle content such as proportional reasoning, algebraic thinking, and geometry and measurement—much sooner than traditional workbooks suggest.

5. My students are math phobic. I don’t know if they can learn math.
   I wonder if they’re afraid of the memories that are conjured up every time they are introduced to math using the same methods that were used when they were in grade school and didn’t “get it.” Have you ever noticed that the topics for all adult education workbooks look very similar—and they all follow the traditional scope and sequence? Students keep getting the same content, taught the same way. And if, year after year, they didn’t understand what was taught, no wonder they’re beginning to dread math!

   If you think your students are math phobic, then it’s critical that you introduce math in friendly ways that allow them many opportunities for success. Students, whether they want to admit it or not, often use math. Build on what they know and understand. Don’t jump from benchmark numbers to messy ones right after they’ve begun to get comfortable with the concept. Give them lots of time to explore concepts using just benchmark numbers (including fractions, decimals, and percents). The EFF Use Math performance continuum highlights what benchmark numbers to use first, including what strategies should be comfortable for students beginning to grapple with math content.

6. I wasn’t trained as a math teacher. I need the structure of a workbook.
   Structure is one thing; holding on to traditional ways of doing things when you know they weren’t necessarily successful is another. If you take the attitude of being a learner yourself, you’re more likely to do more for your students than hanging on to the structure now used. Pose dilemmas for students (and yourself), then figure out new strategies to address the dilemmas. It’s amazing how creative our students can be when given a problem that can be solved in many different ways—and you might be surprised at how much you can learn from your students.

   The EFF Standard Use Math to Solve Problems and Communicate does provide you with some structure. The EFF Use Math performance level descriptors provide additional structure. And, the EFF Teaching/Learning Cycle is a useful organizing tool to help you and your students develop meaningful lessons.

7. Are there materials available that go along with the EFF Use Math performance continuum?
   No, not yet. But there is the EFF Teaching/Learning Cycle that you can use as an organizing tool to create your own activities—with your students’ input. Students’ lives should be the basis for the curriculum, not a workbook.

   The EFF Toolkit
   http://eff.cls.utk.edu/toolkit/index.htm
   and the Guides to Using the Performance Continua
   http://eff.cls.utk.edu/assessment/guides.htm
   have more examples and tools.
Guide to Using the *Use Math to Solve Problems and Communicate* Performance Continuum The purpose of this guide is to introduce the performance continuum for the EFF Standard Use Math to Solve Problems and Communicate and to show you how to use it to plan instruction and classroom-based assessment.

*Site address:* [http://eff.cls.utk.edu/assessment/math1.htm](http://eff.cls.utk.edu/assessment/math1.htm)

The EFF Toolkit Examples for *Use Math to Solve Problems and Communicate* describe math lessons and provide links to tools and supports for these lessons.

*Site address:* [http://eff.cls.utk.edu/toolkit/examples.htm](http://eff.cls.utk.edu/toolkit/examples.htm)

A Framework for Adult Numeracy Standards: The Mathematical Skills and Abilities Adults Need to Be Equipped for the Future. This research project, funded by the National Institute for Literacy, informed the EFF Standard *Use Math to Solve Problems and Communicate*.


Developing Adults’ Numerate Thinking: Getting Out from Under the Workbooks (*Focus on Basics*: V4B, Sept. 2000). This article suggests that we seriously reflect on what and how we presently teach “math” to adults.


The Effects of Continuing Goal-Setting on Persistence in a Math Classroom (*Focus on Basics*: V4A, March 2000). An adult education math instructor shares the results of her own research on the effects of goal-setting.


The LINCS Science and Numeracy Collection, organized by the four major strands: number sense, data and statistics, spatial sense and measurement, algebraic thinking, as well as another strand: problem-solving.

*Site address:* [http://literacynet.org/sciencelincs/home.html](http://literacynet.org/sciencelincs/home.html)

The Math Forum homepage contains a wealth of resources on math education. While there is only a limited number of resources specific to adult numeracy, it does provide a variety of activities and articles on topics that are relevant to adults as well as all other age groups.

*Site address:* [http://mathforum.org](http://mathforum.org)

EMPower Math: The Empower materials, which include a series of eight books, have been developed for and field tested in adult education classrooms by TERC. EMPower is published by Key College Press and is also available from [http://www.peppercornbooks.com/catalog/](http://www.peppercornbooks.com/catalog/)
